Strong Decays of the Radial Excited States $B(2S)$ and $D(2S)$

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ABSTRACT

The strong OZI allowed decays of the first radial excited states $B(2S)$ and $D(2S)$ are studied in the instantaneous Bethe-Salpeter method, and by using these OZI allowed channels we estimate the full decay widths: $\Gamma_{B^0(2S)} = 24.4$ MeV, $\Gamma_{B^+(2S)} = 23.7$ MeV, $\Gamma_{D^0(2S)} = 11.3$ MeV and $\Gamma_{D^+(2S)} = 11.9$ MeV. We also predict the masses of them: $M_{B^0(2S)} = 5.777$ GeV, $M_{B^+(2S)} = 5.774$ GeV, $M_{D^0(2S)} = 2.390$ GeV and $M_{D^+(2S)} = 2.393$ GeV.

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In the past few years, there are many new states observed in experiments. Among them, the new states $D_{s0}^*(2317)$, $D_{s1}(2460)$ [1], $B_{s1}(5830)$ and $B_{s2}(5840)$ [2] are orbitally excited states, which are also called P wave states. So far, great progress has been made on the physics of orbital excited states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ [3], and there are already exist some investigations of $B_{s1}(5830)$ and $B_{s2}(5840)$ [4]. Around the energy of these hadrons, according to constitute quark model, there may be the radial excited S wave states $B(2S)$ and $D(2S)$. But due to their absence, the experimental and theoretical studies for the radial excited $2S$ states $B(2S)$ and $D(2S)$ are still missing in the literature.

We know that the first radial excited $2S$ state has a node structure in its wave function, which means relativistic correction of $2S$ state is much larger than the one of corresponding basic state, even the $2S$ state is a heavy meson, so to consider the physics of radial excited state a relativistic method is needed. Bethe-Salpeter equation [5] and its instantaneous one, Salpeter equation [6], are famous relativistic methods to describe the dynamics of a bound state. In a previous letter [7], we have solved the full Salpeter equations for pseudoscalar mesons, the masses of first radial excited $2S$ states are obtained, they are $M_{B^0(2S)} = 5.777$ GeV, $M_{B^+(2S)} = 5.774$ GeV, $M_{D^0(2S)} = 2.390$ GeV and $M_{D^+(2S)} = 2.393$ GeV.

The mass of $B(2S)$ is 310 MeV higher than the threshold of mass scale of $B^*\pi$, but lower than the threshold of B_s^*K, and the mass of $D(2S)$ is 240 MeV higher than the threshold of mass scale of $D^*\pi$, but lower than the threshold of D_s^*K, so the strong decays $B(2S) \to B^* + \pi$ and $D(2S) \to D^* + \pi$ are OZI allowed strong decays, and they are dominate decay channels of $B(2S)$ and $D(2S)$, respectively. In this letter, we calculate the strong decay widths of $B(2S) \to B^* + \pi$ and $D(2S) \to D^* + \pi$ in the framework of Bethe-Salpeter method.

Since one of the final state is π meson in the OZI allowed $B(2S)$ or $D(2S)$ strong decay, we use the reduction formula, PCAC relation and low energy theorem, so for the strong decays (considering the $B^0(2S) \to B^{*+}\pi^-$ as an example) shown in Fig. 1, the transition matrix element can be written as [8]:

$$T = \frac{P_{f_2}^{\mu}}{f_{P_{f_2}}} \langle B^{*+}(P_{f_1})|\bar{u}\gamma_\mu\gamma_5 d|B^0(P)\rangle, \tag{1}$$

where P, P_{f_1} and P_{f_2} are the momenta of the initial state $B^0(2S)$, final states B^{*+} and π^-, respectively, and $f_{P_{f_2}}$ is the decay constant of π^- meson.

To evaluate Eq. (1), we need to calculate the hadron matrix element $\langle B^{*+}(P_{f_1})|\bar{u}\gamma_\mu\gamma_5 d|B^0(P)\rangle$. It is well known that the Mandelstam formalism [9] is one of proper approaches to compute the hadron matrix elements sandwiched by the Bethe-Salpeter or Salpeter wave functions of two bound-state. With

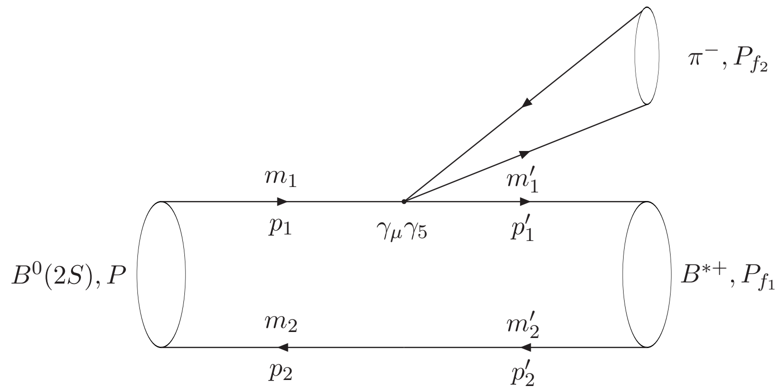

Figure 1: Feynman diagram corresponding to the strong decays $B^0(2S) \to B^{*+}\pi^-$.

the help of this method, in leading order, the hadron matrix elements in the center of mass system of initial meson can be written as [8, 10]:

$$\langle B^{*+}(P_{f_1})|\bar{u}\gamma_\mu\gamma_5 d|B^0_{2S}(P)\rangle = \int \frac{d\vec{q}}{(2\pi)^3} Tr\left[\bar{\varphi}^{++}_{P_{f_1}}(\vec{q}')\gamma_\mu\gamma_5\varphi^{++}_{P}(\vec{q})\frac{\not{P}}{M}\right]. \tag{2}$$

where $\vec{q}$ is the relative three-momentum of the quark-anti-quark in the initial meson $B^0(2S)$ and $\vec{q}' = \vec{q} + \frac{m_2'}{m_1'+m_2'}\vec{r}$, M is the mass of $B^0(2S)$, $\vec{r}$ is the three dimensional momentum of the final meson B^{*+}, φ^{++}_{P} is the positive energy B.S. wave function for the relevant mesons and $\bar{\varphi}^{++}_{P_{f_1}} = \gamma_0(\varphi^{++}_{P_{f_1}})^{+}\gamma_0$.

For the initial state pseudoscalar meson $B^0(2S)$ ($J^P = 0^-$), the positive energy wave function takes the general form [7]:

$$\begin{aligned}\varphi^{++}_{0^-}(\vec{q}) &= \frac{M}{2}\left\{\left[f_1(\vec{q}) + f_2(\vec{q})\frac{m_1+m_2}{\omega_1+\omega_2}\right]\left[\frac{\omega_1+\omega_2}{m_1+m_2} + \frac{\not{P}}{M} - \frac{\not{q}_\perp(m_1-m_2)}{m_2\omega_1+m_1\omega_2}\right]\right.\\ &+ \left.\frac{\not{q}_\perp\not{P}(\omega_1+\omega_2)}{M(m_2\omega_1+m_1\omega_2)}\right\}\gamma_5.\end{aligned} \tag{3}$$

where $q_\perp = (0, \vec{q})$ and $\omega_i = \sqrt{m_i^2 + \vec{q}^2}$, $f_i(\vec{q})$ are eigenvalue wave functions which can be obtained by solving the full 0^- state Salpeter equations. For the final state vector meson B^{*+} ($J^P = 1^-$), the positive energy wave function takes the general form [11]:

$$\begin{aligned}\varphi^{++}_{1^-}(\vec{q}') &= \frac{1}{2}\left[A\,\not{\epsilon}'^{\lambda}_{\perp} + B\,\not{\epsilon}'^{\lambda}_{\perp}\,\not{P}_{f_1} + C(\not{q}'_\perp\,\not{\epsilon}'^{\lambda}_{\perp} - q'_\perp\cdot\epsilon'^{\lambda}_{\perp}) + D(\not{P}_{f_1}\,\not{\epsilon}'^{\lambda}_{\perp}\,\not{q}'_\perp - \not{P}_{f_1}q'_\perp\cdot\epsilon'^{\lambda}_{\perp})\right.\\ &+ \left. q'_\perp\cdot\epsilon'^{\lambda}_{\perp}(E + F\,\not{P}_{f_1} + G\,\not{q}'_\perp + H\,\not{P}_{f_1}\,\not{q}'_\perp)\right],\end{aligned} \tag{4}$$

where ϵ is the polarization vector of meson, and $A,\ B,\ C,\ D,\ E,\ F,\ G,\ H$ are defined as:

$$\begin{aligned}A &= M'\left[f_5(\vec{q}') - f_6(\vec{q}')\frac{\omega_1'+\omega_2'}{m_1'+m_2'}\right],\\ B &= \left[f_6(\vec{q}') - f_5(\vec{q}')\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right],\end{aligned}$$

$$
\begin{aligned}
C &= \frac{M'(\omega_2'-\omega_1')}{m_2'\omega_1'+m_1'\omega_2'}\left[f_5(\vec{q'})-f_6(\vec{q'})\frac{\omega_1'+\omega_2'}{m_1'+m_2'}\right],\\
D &= \frac{\omega_1'+\omega_2'}{\omega_1'\omega_2'+m_1'm_2'+\vec{q'}^2}\left[f_5(\vec{q'})-f_6(\vec{q'})\frac{\omega_1'+\omega_2'}{m_1'+m_2'}\right],\\
E &= \frac{m_1'+m_2'}{M'(\omega_1'\omega_2'+m_1'm_2'-\vec{q'}^2)}\left\{M'^2\left[f_5(\vec{q'})-f_6(\vec{q'})\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right]-\vec{q'}^2\left[f_3(\vec{q'})+f_4(\vec{q'})\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right]\right\},\\
F &= \frac{\omega_1'-\omega_2'}{M'^2(\omega_1'\omega_2'+m_1'm_2'-\vec{q'}^2)}\left\{M'^2\left[f_5(\vec{q'})-f_6(\vec{q'})\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right]-\vec{q'}^2\left[f_3(\vec{q'})+f_4(\vec{q'})\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right]\right\},\\
G &= \left\{\frac{1}{M'}\left[f_3(\vec{q'})+f_4(\vec{q'})\frac{m_1'+m_2'}{\omega_1'+\omega_2'}\right]-\frac{2f_6(\vec{q'})M'}{m_2'\omega_1'+m_1'\omega_2'}\right\},\\
H &= \frac{1}{M'^2}\left\{\left[f_3(\vec{q'})\frac{\omega_1'+\omega_2'}{m_1'+m_2'}+f_4(\vec{q'})\right]-2f_5(\vec{q'})\frac{M'^2(\omega_1'+\omega_2')}{(m_1'+m_2')(\omega_1'\omega_2'+m_1'm_2'+\vec{q'}^2)}\right\}.
\end{aligned}
\tag{5}
$$

where M' is the mass of B^{*+}, eigenvalue wave functions $f_i(\vec{q'})$ can be obtained by solving the full 1^- state Salpeter equations.

In calculation of transition matrix element and solving the full Salpeter equation, there are some parameters have to be fixed, the input parameters are chosen as follows [7]: $m_b = 5.224$ GeV, $m_c = 1.7553$ GeV, $m_d = 0.311$ GeV, $m_u = 0.307$ GeV. The values of the decay constants we use in this letter are $f_{\pi^\pm} = 0.1307$ GeV, $f_{\pi^0} = 0.13$ GeV [12]. With the parameters, the masses of the radial excited $2S$ states are present: $M_{B^0(2S)} = 5.777$ GeV, $M_{B^+(2S)} = 5.774$ GeV, $M_{D^0(2S)} = 2.390$ GeV and $M_{D^+(2S)} = 2.393$ GeV. The numerical strong decay widths of $B(2S)$ and $D(2S)$ mesons are shown in Table 1.

In our results only the 1^-0^- final states are calculated ($B^*\pi$ and $D^*\pi$), in our estimate of mass spectra, there are no other OZI allowed strong decay channels. For example, from the analysis of quantum number, there may be the decay channels with P wave in the final states, for example, the final state can be 0^+0^- $B(1P)\pi$ states, but due to our estimate the mass of lightest P wave 0^+ state $m_{B(1P)} = 5.665$ GeV [13], which is larger than the threshold of $B(2S)$ (the same results for $D(2S)$ cases), so there is no phase space for this channel, if later experimental discovery of mass of this state is lower than theoretical estimate like happened to $D_{s0}(2317)$, which has been hoped much higher than 2317 MeV, this channel become a OZI allowed one, but because the phase space is very small, and it is a P wave, the transition decay width should be smaller than the case when it is S wave, so we can ignore the contributions of these channels and other electroweak channels, and we use these OZI allowed decay widths to estimate the full decay width of this $2S$ state.

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Table 1: The strong decay widths of the $2S$ state B and D mesons.

Mode	Γ (MeV)	Mode	Γ (MeV)
$B^0(2S) \to B^{*+}\pi^-$	12.3	$D^0(2S) \to D^{*+}\pi^-$	5.48
$B^0(2S) \to B^{*0}\pi^0$	12.1	$D^0(2S) \to D^{*0}\pi^0$	5.85
$B^+(2S) \to B^{*0}\pi^+$	11.7	$D^+(2S) \to D^{*0}\pi^+$	6.05
$B^+(2S) \to B^{*+}\pi^0$	12.0	$D^+(2S) \to D^{*+}\pi^0$	5.80

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